MIDTERM EXAMINATION

FORM

A

1. The following function, f, maps the real line into itself. $f: R \to R$. We must decide if it is continuous at x = 0. Please choose between two proposed answers below. Explain your choice.

For $x < 0, f(x) = x$
$x = 0, f(x) = 0$
$x > 0, f(x) = 2x$

Proposed answer A: *Yes the function is continuous at 0. For any* $\varepsilon > 0$, *choose* $\delta = .3\varepsilon$. *Then for any x in the neighborhood of 0,* $|0 - x| < \delta$, $|f(x) - f(0)| \leq |2x - 0| < 2\delta = .6\varepsilon < \varepsilon$. *This is the definition of continuity.*

Proposed answer B: *No the function is not continuous at 0. It makes a jump in the neighborhood of* $x > 0$. *For* $2 > \varepsilon > 0$, *there is no* $\delta > 0$, *such that for* $x > 0$ *with* $|x - 0| < \delta$, *it follows that* $|f(x) - f(0)| < \varepsilon$. *Instead we have* $|f(x) - f(0)| \geq 2 > \varepsilon$.

Problem 2 below treats allocation in an Edgeworth Box. There are two households, denoted 1 and 2, two goods denoted x and y. The two households have identical preferences denoted by the utility function U(x, y) = x · y . For any consumption plan (x, y), with x and y > 0, the household's marginal rate of substitution,

$$\mathbf{MRS_{x,y}} = \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}} = \frac{U_x}{U_y} = \frac{y}{x}.$$

Household 1's endowment is $r^1 = (60,0)$; household 2's endowment is $r^2 = (60, 60)$.

2. The Walrasian auctioneer proposes the price vector

$p = (p_x, p_y) = (^1/_3, ^2/_3)$. In order for this to be a competitive equilibrium price vector there must be a consumption plan $(x^1, y^1), (x^2, y^2)$ for each household that fulfills budget constraint, optimizes utility subject to budget constraint, and clears the market. Consider $(x^1, y^1) = (30, 15)$, $(x^2, y^2) = (90, 45)$. Demonstrate that this allocation is a competitive equilibrium.

3. The Brouwer Fixed Point Theorem can be applied to the closed unit interval in R, [0, 1] = {x | x $\in$ R, 0 $\leq$ x $\leq$ 1}. Then we say, *Let f:[0, 1] $\rightarrow$ [0, 1], f continuous on [0, 1], then there is x* in [0, 1] so that f(x*) = x*.*

Demonstrate that this statement is not true (that is, the fixed point may not exist) when we revise the domain and range to the open unit interval (0, 1) = {x | x $\in$ R, 0 < x < 1}. That is, show that the following statement is false: *Let f(0, 1) $\rightarrow$ (0, 1), f continuous on (0, 1), then there is x* in (0, 1) so that f(x*) = x*.*

A counterexample with an explanation is sufficient.

Hint: Consider $f(x) = .5 + .5x$. $f: (0,1) \rightarrow (0,1)$. Use a proof by contradiction. Suppose there is $x^* = f(x^*) = .5 + .5x^*$. Then $.5x^* = .5$, and $x^*=1$. Is this a contradiction? Explain.

4. A subset S of $\mathbf{R}^{\mathbf{N}}$ is said to be "closed" if it contains all its limit points --- that is, if every convergent sequence in S has its limit point in S. A subset S of $\mathbf{R}^{\mathbf{N}}$ is said to be “bounded” if it can be contained in a cube (centered at the origin) with a side of finite length. A subset S of $\mathbf{R}^{\mathbf{N}}$ is said to be “convex,” if --- for every choice of two point of S ---- the line segment connecting the two points is contained in S.

Consider $\mathbf{R}^{\mathbf{N}}$ as a subset of $\mathbf{R}^{\mathbf{N}}$.

(a) Is $\mathbf{R}^{\mathbf{N}}$ closed? Give an explanation of your answer, or an example illustrating it.

(b) Is $\mathbf{R}^{\mathbf{N}}$ bounded? Give an explanation of your answer, or an example illustrating it.

(c) Is $\mathbf{R}^{\mathbf{N}}$ convex? Give an explanation of your answer, or an example illustrating it.