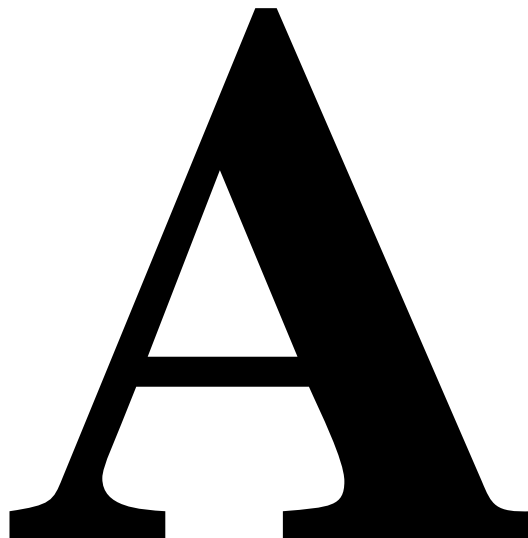


MIDTERM EXAMINATION

FORM



1. The following function, f , maps the real line into itself. $f: \mathbb{R} \rightarrow \mathbb{R}$. We must decide if it is continuous at $x = 0$. Please choose between two proposed answers below. Explain your choice.

$$\begin{aligned} \text{For } x < 0, f(x) &= x \\ x = 0, f(x) &= 0 \\ x > 0, f(x) &= 2x \end{aligned}$$

Proposed answer A: Yes the function is continuous at 0. For any $\varepsilon > 0$, choose $\delta = .3\varepsilon$. Then for any x in the neighborhood of 0, $|0 - x| < \delta$, $|f(x) - f(0)| \leq |2x - 0| < 2\delta = .6\varepsilon < \varepsilon$. This is the definition of continuity.

Proposed answer B: No the function is not continuous at 0. It makes a jump in the neighborhood of $x > 0$. For $2 > \varepsilon > 0$, there is no $\delta > 0$, such that for $x > 0$ with $|x - 0| < \delta$, it follows that $|f(x) - f(0)| < \varepsilon$. Instead we have $|f(x) - f(0)| \geq 2 > \varepsilon$.

Problem 2 below treats allocation in an Edgeworth Box. There are two households, denoted 1 and 2, two goods denoted x and y . The two households have identical preferences denoted by the utility function $U(x, y) = x \cdot y$. For any consumption plan (x, y) , with x and $y > 0$, the household's marginal rate of substitution,

$$\text{MRS}_{x,y} = \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}} = \frac{U_x}{U_y} = \frac{y}{x}.$$

Household 1's endowment is $r^1 = (60, 0)$; household 2's endowment is $r^2 = (60, 60)$.

2. The Walrasian auctioneer proposes the price vector

$p = (p_x, p_y) = (1/3, 2/3)$. In order for this to be a competitive equilibrium price vector there must be a consumption plan $(x^1, y^1), (x^2, y^2)$ for each household that fulfills budget constraint, optimizes utility subject to budget constraint, and clears the market. Consider $(x^1, y^1) = (30, 15)$, $(x^2, y^2) = (90, 45)$. Demonstrate that this allocation is a competitive equilibrium.

3. The Brouwer Fixed Point Theorem can be applied to the closed unit interval in \mathbf{R} , $[0, 1] = \{x \mid x \in \mathbf{R}, 0 \leq x \leq 1\}$. Then we say, *Let $f: [0, 1] \rightarrow [0, 1]$, f continuous on $[0, 1]$, then there is x^* in $[0, 1]$ so that $f(x^*) = x^*$.*

Demonstrate that this statement is not true (that is, the fixed point may not exist) when we revise the domain and range to the open unit interval $(0, 1) = \{x \mid x \in \mathbf{R}, 0 < x < 1\}$. That is, show that the following statement is false: *Let $f: (0, 1) \rightarrow (0, 1)$, f continuous on $(0, 1)$, then there is x^* in $(0, 1)$ so that $f(x^*) = x^*$.*

A counterexample with an explanation is sufficient.

Hint: Consider $f(x) = .5 + .5x$. $f: (0, 1) \rightarrow (0, 1)$. Use a proof by contradiction. Suppose there is $x^* = f(x^*) = .5 + .5x^*$. Then $.5x^* = .5$, and $x^* = 1$. Is this a contradiction? Explain.

4. A subset S of \mathbf{R}^N is said to be "closed" if it contains all its limit points --- that is, if every convergent sequence in S has its limit point in S . A subset S of \mathbf{R}^N is said to be "bounded" if it can be contained in a cube (centered at the origin) with a side of finite length. A subset S of \mathbf{R}^N is said to be "convex," if --- for every choice of two points of S --- the line segment connecting the two points is contained in S .

Consider \mathbf{R}^N as a subset of \mathbf{R}^N .

(a) Is \mathbf{R}^N closed? Give an explanation of your answer, or an example illustrating it.

(b) Is \mathbf{R}^N bounded? Give an explanation of your answer, or an example illustrating it.

(c) Is \mathbf{R}^N convex? Give an explanation of your answer, or an example illustrating it.